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Structural Equation Modeling using Stata

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Abstract. In this tutorial, you will learn how to fit structural equation models (SEM) using Stata software. SEMs can be fit in Stata using the `sem` command for standard linear SEMs, the `gsem` command for generalized linear SEMs, or by drawing their path diagrams in the SEM Builder. After a brief introduction to Stata, the `sem` command will be demonstrated through a confirmatory factor analysis model, mediation model, group analysis, and a growth curve model, and the `gsem` command will be demonstrated through a random-slope model and a logistic ordinal regression. Materials and datasets are provided online, allowing anyone with Stata to follow along.

Keywords: Structural Equation Modeling · Growth Curve Modeling · Mediation · Software.

1 Introduction

Structural equation modeling (SEM) is a multivariate statistical analysis framework that allows simultaneous estimation of a system of equations. SEM can be used to fit a wide range of models, including those involving measurement error and latent constructs. This tutorial will demonstrate how to fit a variety of SEMs using Stata statistical software (StataCorp, 2021). Specifically, we will fit models in Stata with both measurement and structural components, as well as those with random effects and generalized responses. We will assess model fit, compute modification indices, estimate mediation effects, conduct group analysis, and more. First, however, we will begin with an introduction to Stata itself. Familiarity with SEM theory and concepts is assumed.

Stata is a complete, integrated software package that provides tools for data manipulation, visualization, statistics, and automated reporting. The Data Editor, Variables window, and Properties window can be used to view and edit your dataset and to manage variables, including their names, labels, value labels, notes, formats, and storage types. Commands can be typed into the Command window, or generated through the point-and-click interface. Log files keep a record of every command issued in a session, while do-files save selected commands to allow users to replicate their work. To learn more about a command, you can type help followed by the command name in the Command window and the Viewer window will open with the help file and provide links to further documentation. Stata's documentation consists of over 17,000 pages detailing each feature in Stata including the methods and formulas and fully worked examples.

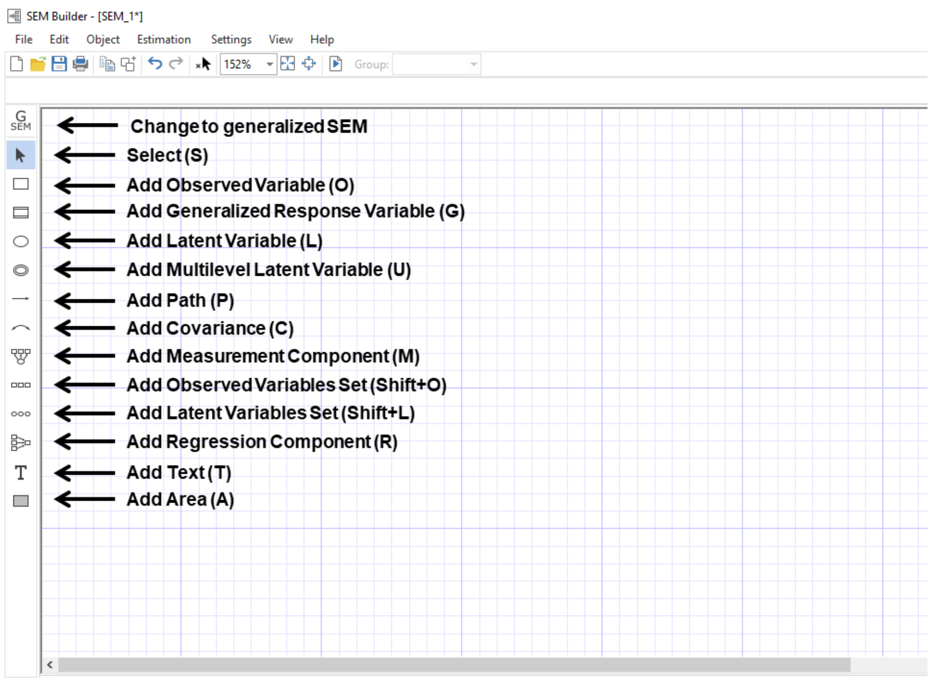


Figure 1. SEM Builder

There are three ways to fit SEMs in Stata: the sem command, the gsem command, and through the SEM Builder. The sem command is for fitting standard linear SEMs. It is quicker and has more features for testing and interpreting results than gsem. The gsem command is for fitting models with generalized responses, such as binary, count, or categorical responses, models with random effects, and mixture models. Both sem and gsem models can be fit via path diagrams using the SEM Builder. You can open the SEM Builder window by typing sembuilder into the Command window. See the interface in Figure 1; click the tools you need on the left, or type their shortcuts shown in the parentheses. To fit gsem models, the GSEM button must first be selected. Estimation and diagram settings can be changed using the menus at the top. The Estimate button fits the model. Path diagrams can be saved as .stsem files to be modified later, or can be exported to a variety of image formats (for example see Figure 2). Although this tutorial will focus on the sem and gsem commands, the Builder shares the same functionality. You can watch a demonstration with the SEM Builder on the StataCorp YouTube Channel: `https://www.youtube.com/watch?v=HeQcha3C8Fk`

To download the datasets, do-file, and path diagrams, you can type the following into Stata's Command window:

```
. net from http://www.stata.com/users/mcain/JBDS_SEM
```

Clicking on the `SEMtutorial` link will download the materials to your current working directory. To open the do-file with the commands we'll be using, you can type

```
. doedit SEMtutorial
```

Commands can either be executed from the do-file or typed into the Command window. We'll start by loading and exploring our first dataset. These data contain observations on four indicators for socioeconomic status of high school students as well as their math scores, school types (private or public), and the student-teacher ratio of their school. Alternatively, we could have used a summary statistics dataset containing means, variances, and correlations of the variables rather than observations.

```
. use math

. codebook, compact

Variable   Obs Unique      Mean  Min  Max  Label
------------------------------------------------------------------------
schtype    519      2    .61079    0    1  School type
ratio      519     14  16.75723   10   28  Student-Teacher ratio
math       519     42  51.72254   30   71  Math score
ses1       519      5  1.982659    0    4  SES item 1
ses2       519      5  2.003854    0    4  SES item 2
ses3       519      5  2.003854    0    4  SES item 3
ses4       519      5  2.003854    0    4  SES item 4
------------------------------------------------------------------------
```

2 Fitting models with the `sem` command

2.1 Path Analysis

Let's start our analysis by fitting the one-factor confirmatory factor analysis (CFA) model shown in Figure 2. Using the `sem` command, paths are specified in parentheses and the direction of the relationships are specified using arrows, i.e. `(x->y)`. Arrows can point in either direction, `(x->y)` or `(y<-x)`. Paths can be specified individually, or multiple paths can be specified within a single set of parentheses, `(x1 x2 x3 -> y)`. By default, Stata assumes that all lower-case variables are observed and uppercase variables are latent. You can change these settings using the `nocapslatent` and the `latent()` options. In Stata, options are always added after a comma. We'll see plenty of examples of this later.

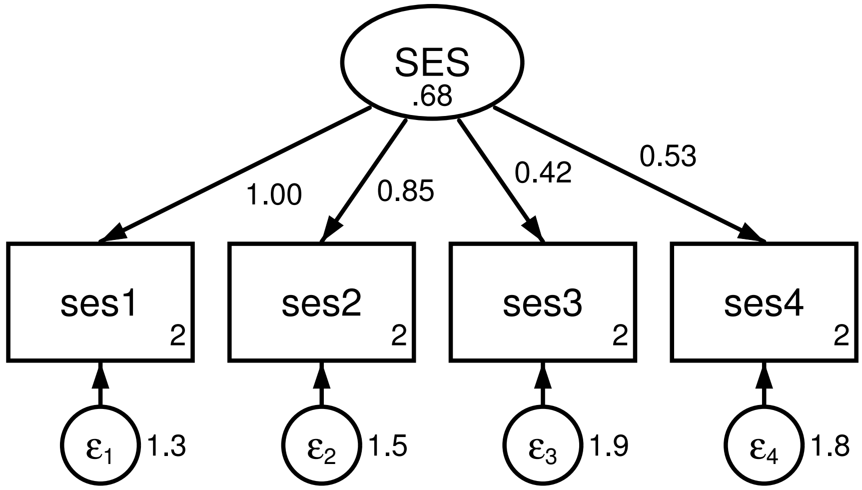


Figure 2. One-factor CFA measuring socioeconomic status (SES)

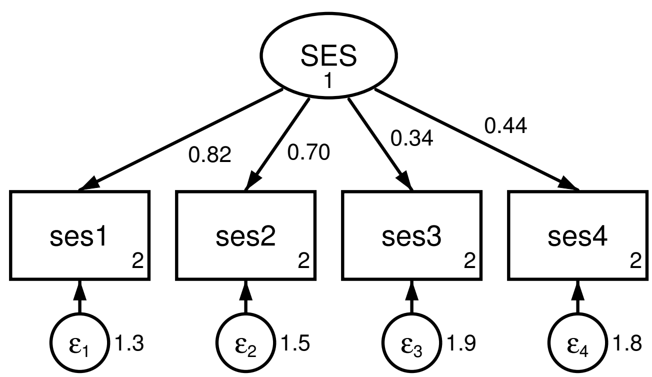
```
. sem (SES -> ses1-ses4)
Endogenous variables
  Measurement: ses1 ses2 ses3 ses4
Exogenous variables
  Latent: SES
Fitting target model:
Iteration 0:   log likelihood = -3621.9572
Iteration 1:   log likelihood = -3621.5801
Iteration 2:   log likelihood = -3621.5573
Iteration 3:   log likelihood =  -3621.557
Structural equation model                              Number of obs = 519
Estimation method: ml
Log likelihood = -3621.557
 ( 1)  [ses1]SES = 1
```

	Coefficient	OIM std. err.	z	P>|z|	[95% conf.	interval]
Measurement						
ses1						
SES	1	(constrained)				
_cons	1.982659	.0620424	31.96	0.000	1.861058	2.10426
ses2						
SES	.8481035	.1962358	4.32	0.000	.4634884	1.232719
_cons	2.003854	.0620169	32.31	0.000	1.882303	2.125404
ses3						
SES	.416385	.1331306	3.13	0.002	.1554539	.6773161
_cons	2.003854	.062017	32.31	0.000	1.882302	2.125405
ses4						

SES	.5315065	.1517342	3.50	0.000	.234113	.8289001
_cons	2.003854	.062017	32.31	0.000	1.882302	2.125405
var(e.ses1)	1.317579	.1855509			.9997798	1.736397
var(e.ses2)	1.506881	.1493285			1.240872	1.829916
var(e.ses3)	1.878203	.1257611			1.647204	2.141595
var(e.ses4)	1.803979	.1287389			1.568507	2.074801
var(SES)	.6801844	.1908617			.3924434	1.178898

```
LR test of model vs. saturated: chi2(2) = 11.03          Prob > chi2 = 0.0040
```

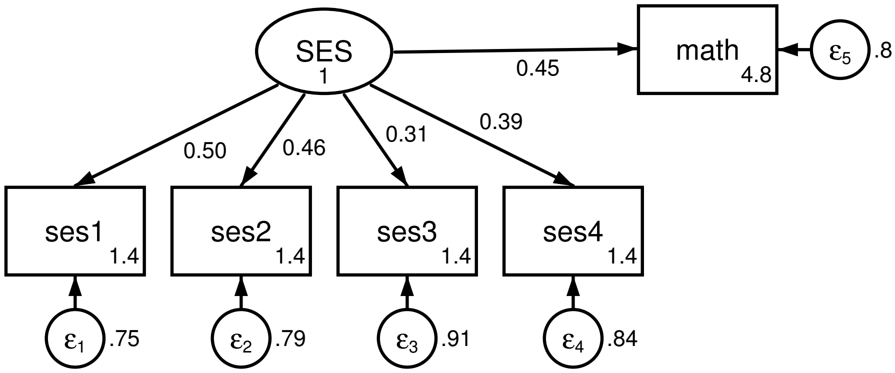
Viewing the results, we see that by default Stata constrained the first factor loading to be 1 and estimated the variance of the latent variable. If, instead, we would like to constrain the variance and estimate all four factor loadings, we could use the `var()` option. Constraints in any part of the model can be specified using the `@` symbol. To save room, syntax and results for this and the remaining models will be shown on their path diagrams; see Figure 3.




```
. sem (SES -> ses1-ses4), var(SES@1)
```

Figure 3. One-factor CFA with constrained variance.

Specifying structural paths is no different from specifying measurement paths. We can add math score to our model and hypothesize that socioeconomic status influences expected math performance. This model is shown in Figure 4; we've added the `standardized` option to get standardized coefficients. With every increase of one standard deviation in SES, math score is expected to increase by 0.45 standard deviations.




```
. sem (SES -> ses1-ses4 math), standardized
```

Figure 4. SES influences math scores.

To get fit indices for our model, we can use the postestimation command `estat gof` after any `sem` model. Add the `stats(all)` option to see all fit indices.

```
. estat gof, stats(all)
```

Fit statistic	Value	Description
Likelihood ratio		
chi2_ms(5)	17.689	model vs. saturated
p > chi2	0.003	
chi2_bs(10)	150.126	baseline vs. saturated
p > chi2	0.000	
Population error		
RMSEA	0.070	Root mean squared error of approximation
90% CI, lower bound	0.037	
upper bound	0.107	
pclose	0.147	Probability RMSEA <= 0.05
Information criteria		
AIC	11157.441	Akaike´s information criterion
BIC	11221.219	Bayesian information criterion
Baseline comparison		
CFI	0.909	Comparative fit index
TLI	0.819	Tucker-Lewis index
Size of residuals		
SRMR	0.040	Standardized root mean squared residual
CD	0.532	Coefficient of determination

Satorra-Bentler adjusted model fit indices can be obtained by adding the `vce(sbentler)` option to our model statement and recalculating the model fit indices. This option still uses maximum likelihood estimation, the default, but adjusts the standard errors and the fit indices. Alternatively, estimation can be changed to asymptotic distribution-free or full-information maximum likelihood for missing values using the `method(adf)` or `method(mlmv)` options, respectively. For this example, we'll use the Satorra-Bentler adjustment (Satorra & Bentler, 1994). First, we'll store the current model to use again later.

```
. estimates store m1
. sem (SES -> ses1-ses4 math), vce(sbentler)
Endogenous variables
  Measurement: ses1 ses2 ses3 ses4 math
Exogenous variables
  Latent: SES
Fitting target model:
Iteration 0:   log pseudolikelihood = -5564.2324
Iteration 1:   log pseudolikelihood = -5563.7459
Iteration 2:   log pseudolikelihood = -5563.7204
Iteration 3:   log pseudolikelihood = -5563.7204
Structural equation model                              Number of obs = 519
Estimation method: ml
Log pseudolikelihood = -5563.7204
 ( 1)  [ses1]SES = 1
------------------------------------------------------------------------------
             |              Satorra-Bentler
             | Coefficient  std. err.      z    P>|z|     [95% conf. interval]
-------------+----------------------------------------------------------------
Measurement  |
  ses1       |
         SES |          1  (constrained)
       _cons |   1.982659   .0621024    31.93   0.000     1.860941    2.104377
  -----------+----------------------------------------------------------------
  ses2       |
         SES |   .9278593    .169484     5.47   0.000     .5956767    1.260042
       _cons |   2.003854   .0620769    32.28   0.000     1.882185    2.125522
  -----------+----------------------------------------------------------------
  ses3       |
         SES |    .620192   .1438296     4.31   0.000     .3382912    .9020928
       _cons |   2.003854   .0620769    32.28   0.000     1.882185    2.125522
  -----------+----------------------------------------------------------------
  ses4       |
         SES |   .7954927   .1580751     5.03   0.000     .4856712    1.105314
       _cons |   2.003854   .0620769    32.28   0.000     1.882185    2.125522
  -----------+----------------------------------------------------------------
  math       |
         SES |   6.858402   1.335695     5.13   0.000     4.240488    9.476315
       _cons |   51.72254   .4700825   110.03   0.000      50.8012    52.64389
-------------+----------------------------------------------------------------
  var(e.ses1)|   1.506551   .1203549                        1.2882    1.761913
  var(e.ses2)|   1.573228   .1228219                      1.350014    1.833348
  var(e.ses3)|   1.807189   .0933725                      1.633143    1.999783
  var(e.ses4)|   1.685282   .1047979                      1.491906    1.903724
  var(e.math)|   91.36045   6.594622                       79.3079    105.2447
     var(SES)|   .4912213   .1193158                      .3051572    .7907347
------------------------------------------------------------------------------
LR test of model vs. saturated: chi2(5) = 17.69          Prob > chi2 = 0.0034
Satorra-Bentler scaled test:    chi2(5) = 17.80          Prob > chi2 = 0.0032
```

```
. estat gof, stats(all)

----------------------------------------------------------------------------
Fit statistic        |      Value   Description
---------------------+------------------------------------------------------
Likelihood ratio     |
          chi2_ms(5) |     17.689   model vs. saturated
            p > chi2 |      0.003
         chi2_bs(10) |    150.126   baseline vs. saturated
            p > chi2 |      0.000
                     |
  Satorra-Bentler    |
        chi2sb_ms(5) |     17.804
            p > chi2 |      0.003
       chi2sb_bs(10) |    153.258
            p > chi2 |      0.000
---------------------+------------------------------------------------------
Population error     |
               RMSEA |      0.070   Root mean squared error of approximation
 90% CI, lower bound |      0.037
         upper bound |      0.107
              pclose |      0.147   Probability RMSEA <= 0.05
                     |
  Satorra-Bentler    |
            RMSEA_SB |      0.070   Root mean squared error of approximation
---------------------+------------------------------------------------------
Information criteria |
                 AIC |  11157.441   Akaike's information criterion
                 BIC |  11221.219   Bayesian information criterion
---------------------+------------------------------------------------------
Baseline comparison  |
                 CFI |      0.909   Comparative fit index
                 TLI |      0.819   Tucker-Lewis index
                     |
  Satorra-Bentler    |
              CFI_SB |      0.911   Comparative fit index
              TLI_SB |      0.821   Tucker-Lewis index
---------------------+------------------------------------------------------
Size of residuals    |
                SRMR |      0.040   Standardized root mean squared residual
                  CD |      0.532   Coefficient of determination
----------------------------------------------------------------------------
```

The SB-adjusted CFI is still rather low, 0.91, indicating poor fit. We can use `estat mindices` to compute modification indices that can be used to check for paths and covariances that could be added to the model to improve fit. First, we'll need to restore our original model.

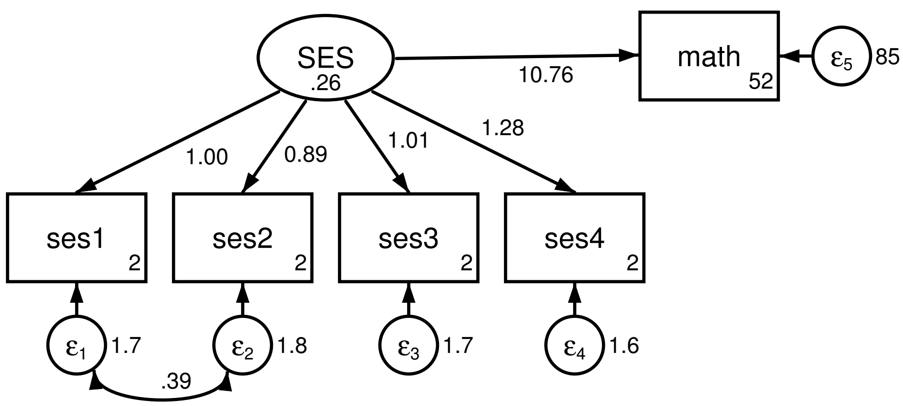
```
. estimates restore m1
```

```
. estat mindices
Modification indices
```

	MI	df	P>MI	EPC	Standard EPC
cov(e.ses1,e.ses2)	16.565	1	0.00	.4818524	.312987
cov(e.ses2,e.ses3)	5.404	1	0.02	-.2203899	-.1307056
cov(e.ses3,e.ses4)	4.956	1	0.03	.2033998	.11655

```
EPC is expected parameter change.
```

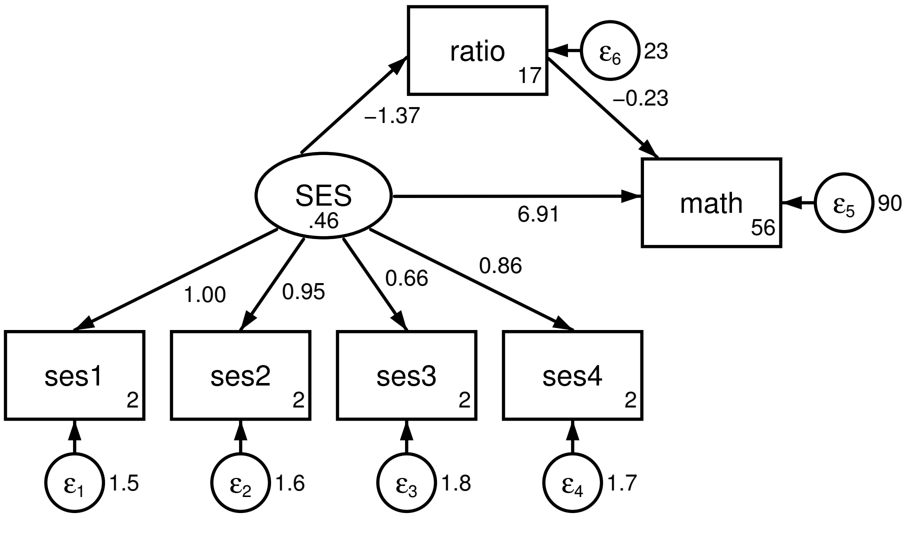
The `MI`, `df`, and `P>MI` are the estimated chi-squared test statistic, degrees of freedom, and p value of the score test testing the statistical significance of the constrained parameter. By default, only parameters that would significantly ($p < 0.05$) improve the model are reported. The `EPC` is the amount that the parameter is expected to change if the constraint is relaxed. According to these results, we see that there is a stronger relationship between the first and second indicator for SES than would be expected given our model, `MI` $= 16.57, p < 0.001$. We could consider adding a residual covariance between these two indicators to our model using the `cov()` option. We use the `e.` prefix to refer to a residual variance of an endogenous variable; see Figure 5.




```
. sem (SES -> ses1-ses4 math), cov(e.ses1*e.ses2)
```

Figure 5. CFA with residual covariance.

One potential explanation of the effect that SES has on math score is that students of higher SES attend schools with smaller student to teacher ratios. We can test this hypothesis using the mediation model shown in Figure 6. Here, we get estimates of the direct effects between each of our variables, but what we would really like to test is the indirect effect between `SES` and `math` through `ratio`. We can get direct effects, indirect effects, and total effects of mediation models with the postestimation command `estat teffects`.




```
. sem (SES -> ses1-ses4 ratio math) (ratio -> math)
```

Figure 6. Student-teacher ratio mediates the effect of SES on math score.

```
. estat teffects
```

Direct effects

	Coefficient	OIM std. err.	z	P>|z|	[95% conf.	interval]
Structural						
ratio						
SES	-1.367306	.5562429	-2.46	0.014	-2.457522	-.2770903
math						
ratio	-.2256084	.1026128	-2.20	0.028	-.4267257	-.024491
SES	6.908564	1.583778	4.36	0.000	3.804417	10.01271
Measurement						
ses1						
SES	1	(constrained)				
ses2						
SES	.9450302	.1643867	5.75	0.000	.6228382	1.267222
ses3						

SES	.6632608	.1725434	3.84	0.000	.3250819	1.00144
ses4						
SES	.8574695	.2012317	4.26	0.000	.4630625	1.251876

Indirect effects

	Coefficient	OIM std. err.	z	P>|z|	[95% conf.	interval]
Structural						
ratio						
SES	0	(no path)				
math						
ratio	0	(no path)				
SES	.3084758	.1451257	2.13	0.034	.0240346	.5929169
Measurement						
ses1						
SES	0	(no path)				
ses2						
SES	0	(no path)				
ses3						
SES	0	(no path)				
ses4						
SES	0	(no path)				

Total effects

	Coefficient	OIM std. err.	z	P>|z|	[95% conf.	interval]
Structural						
ratio						
SES	-1.367306	.5562429	-2.46	0.014	-2.457522	-.2770903
math						
ratio	-.2256084	.1026128	-2.20	0.028	-.4267257	-.024491
SES	7.217039	1.599953	4.51	0.000	4.081189	10.35289
Measurement						
ses1						
SES	1	(constrained)				
ses2						
SES	.9450302	.1643867	5.75	0.000	.6228382	1.267222
ses3						
SES	.6632608	.1725434	3.84	0.000	.3250819	1.00144
ses4						
SES	.8574695	.2012317	4.26	0.000	.4630625	1.251876

In the second group of the output, we see that the mediation effect is not statistically significant, $z = 1.48, p = 0.138$. We may consider bootstrapping this effect to get a more powerful test. We can do this with the `bootstrap` command. First, we need to get labels for the effects we would like to test. We can get these by replaying our model results with the `coeflegend` option. We can use these labels to construct an expression for the mediation effect that we're calling `indirect`. We put this expression in parentheses after `bootstrap` and put any bootstrapping options after a comma; then, we put the model and its options after a colon. Multiple expressions can be included using multiple parentheses sets.

```
. sem, coeflegend

Structural equation model                              Number of obs = 519
Estimation method: ml

Log likelihood = -7117.1959

 ( 1)  [ses1]SES = 1

------------------------------------------------------------------------------
             |  Coefficient  Legend
-------------+----------------------------------------------------------------
Structural   |
  ratio      |
         SES |   -1.367306  _b[ratio:SES]
       _cons |    16.75723  _b[ratio:_cons]
-------------+----------------------------------------------------------------
  math       |
       ratio |   -.2256084  _b[math:ratio]
         SES |    6.908564  _b[math:SES]
       _cons |    55.50311  _b[math:_cons]
-------------+----------------------------------------------------------------
Measurement  |
  ses1       |
         SES |           1  _b[ses1:SES]
       _cons |    1.982659  _b[ses1:_cons]
  -----------+----------------------------------------------------------------
  ses2       |
         SES |    .9450302  _b[ses2:SES]
       _cons |    2.003854  _b[ses2:_cons]
  -----------+----------------------------------------------------------------
  ses3       |
         SES |    .6632608  _b[ses3:SES]
       _cons |    2.003854  _b[ses3:_cons]
  -----------+----------------------------------------------------------------
  ses4       |
         SES |    .8574695  _b[ses4:SES]
       _cons |    2.003854  _b[ses4:_cons]
-------------+----------------------------------------------------------------
  var(e.ses1)|    1.541523  _b[/var(e.ses1)]
  var(e.ses2)|    1.588663  _b[/var(e.ses2)]
  var(e.ses3)|    1.795421  _b[/var(e.ses3)]
  var(e.ses4)|    1.660672  _b[/var(e.ses4)]
 var(e.ratio)|    23.41179  _b[/var(e.ratio)]
  var(e.math)|    89.51067  _b[/var(e.math)]
     var(SES)|    .4562495  _b[/var(SES)]
------------------------------------------------------------------------------
LR test of model vs. saturated: chi2(8) = 21.72           Prob > chi2 = 0.0055
```

```
. bootstrap indirect=(_b[ratio:SES]*_b[math:ratio]), reps(1000) nodots: ///
> sem (SES -> ses1-ses4 ratio math) (ratio -> math)
Bootstrap results                                       Number of obs =   519
                                                        Replications  = 1,000

      Command: sem (SES -> ses1-ses4 ratio math) (ratio -> math)
     indirect: _b[ratio:SES]*_b[math:ratio]
```

	Observed coefficient	Bootstrap std. err.	z	P>|z|	Normal-based [95% conf. interval]	
indirect	.3084758	.1932632	1.60	0.110	-.070313	.6872646

We've added the `reps(1000)` option to compute 1,000 bootstrap replications and the `nodots` option to suppress displaying a dot for each replication. To get 95 percentile confidence intervals based on our bootstrap sampling distribution, we can follow with the postestimation command `estat bootstrap` using the `percentile` option. The resulting confidence interval contains zero so we cannot reject the null hypothesis.

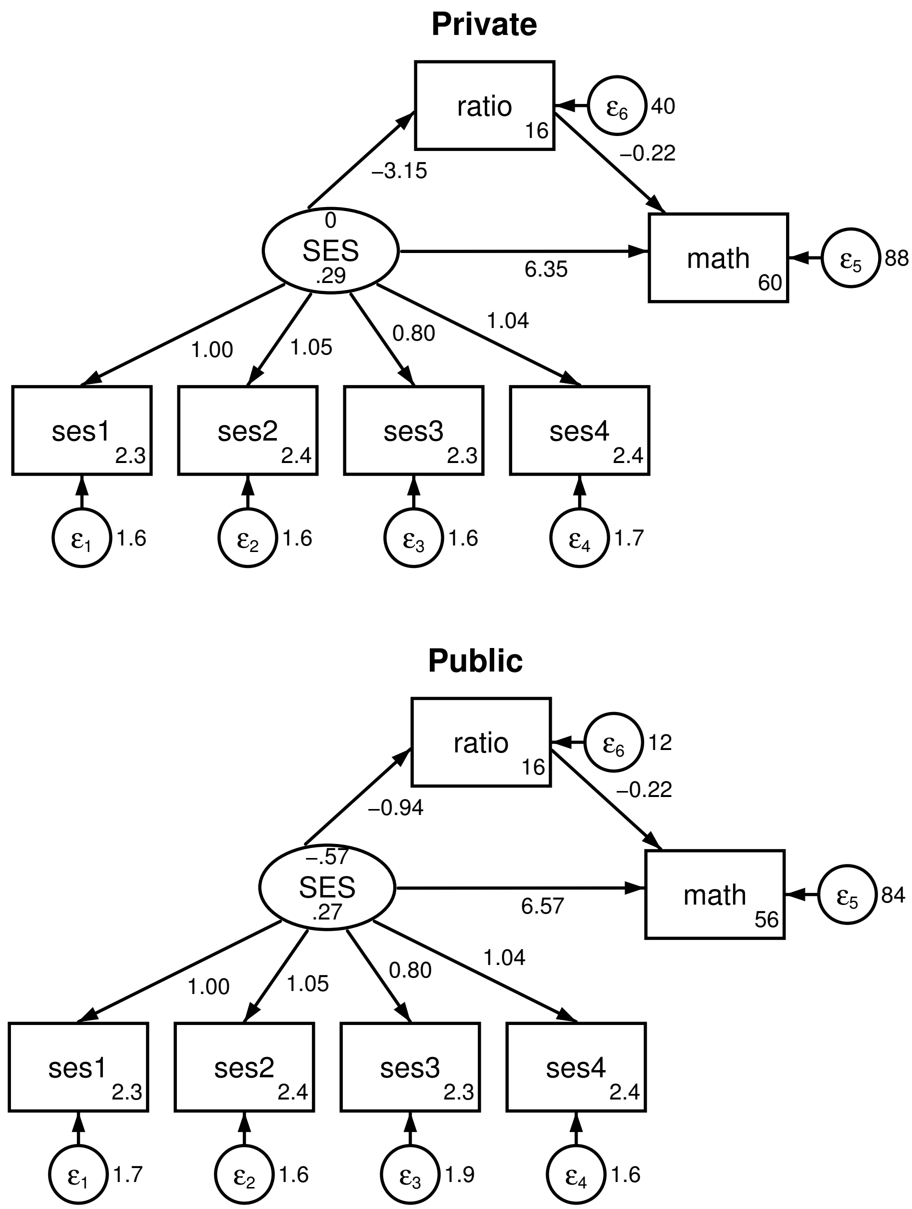
```
. estat bootstrap, percentile
Bootstrap results                               Number of obs     =        519
                                                Replications      =       1000

      Command: sem (SES -> ses1-ses4 ratio math) (ratio -> math)
     indirect: _b[ratio:SES]*_b[math:ratio]
```

	Observed coefficient	Bias	Bootstrap std. err.	[95% conf. interval]		
indirect	.30847577	-.0307326	.19326315	-.0707015	.6837121	(P)

```
Key: P: Percentile
```

2.2 Group Analysis

Finally, we may consider comparing our mediation across groups. Group analysis can be done in Stata by adding the `group()` option. We would like to compare students in public schools versus private schools so we will specify `schtype` as our grouping variable. Then, we can use `ginvariant()` to specify the types of parameters we would like to constrain across groups. All other variables will be estimated separately for each group. The `ginvariant()` options are listed in Table 1. If we don't specify any `ginvariant` option, by default Stata will constrain measurement coefficients and measurement intercepts, `ginvariant(mcoef mcons)`. See the model in Figure 7. Now when we run `estat teffects`, we will get a separate estimated mediation effect for each group.




```
. sem (SES -> ses1-ses4 math) (ratio -> math), group(schtype)
```

Figure 7. Group analysis.

```
. estat teffects, nodirect nototal compact

Indirect effects
------------------------------------------------------------------------------
             |                 OIM
             | Coefficient  std. err.      z    P>|z|     [95% conf. interval]
-------------+----------------------------------------------------------------
Structural   |
  math       |
         SES |
    Private  |   .7043843   .4184641     1.68   0.092    -.1157902    1.524559
     Public  |   .2035724   .1710134     1.19   0.234    -.1316076    .5387525
  -----------+----------------------------------------------------------------
  ratio      |
  -----------+----------------------------------------------------------------
  ses1       |
  -----------+----------------------------------------------------------------
  ses2       |
-------------+----------------------------------------------------------------
Measurement  |
  ses3       |
  -----------+----------------------------------------------------------------
  ses4       |
------------------------------------------------------------------------------
```

Table 1. `ginvariant()` suboptions

Option	Description
`mcoef`	measurement coefficients
`mcons`	measurement intercepts
`merrvar`	covariances of measurement errors
`scoef`	structural coefficients
`scons`	structural intercepts
`serrvar`	covariances of structural errors
`smerrcov`	covariances between structural and measurement errors
`meanex`	means of exogenous variables
`covex`	covariances of exogenous variables
`all`	all the above
`none`	none of the above

To test whether these mediation effects significantly differ, we can conduct a Wald test with the `test` or `testnl` postestimation commands, again using the labels from the `coeflegend` option. Because mediation effects are nonlinear, we will use `testnl`. The mediation effects do not significantly differ between groups, $\chi^2(1) = 1.27, p = 0.260$.

```
. testnl _b[ratio:0bn.schtype#c.SES]*_b[math:0bn.schtype#c.ratio]= ///
> _b[ratio:1.schtype#c.SES]*_b[math:1.schtype#c.ratio]
  (1)  _b[ratio:0bn.schtype#c.SES]*_b[math:0bn.schtype#c.ratio]
> _b[ratio:1.schtype#c.SES]*_b[math:1.schtype#c.ratio]
               chi2(1) =        1.27
           Prob > chi2 =        0.2599
```

```
. estat ginvariant
Tests for group invariance of parameters
```

	Wald test chi2	df	P>chi2	Score test chi2	df	P>chi2
Structural						
math						
ratio	0.001	1	0.9709	.	.	.
SES	0.005	1	0.9441	.	.	.
_cons	1.314	1	0.2516	.	.	.
ratio						
SES	1.825	1	0.1768	.	.	.
_cons	0.011	1	0.9147	.	.	.
Measurement						
ses1						
SES	.	.	.	1.832	1	0.1759
_cons	.	.	.	5.997	1	0.0143
ses2						
SES	.	.	.	0.072	1	0.7882
_cons	.	.	.	0.341	1	0.5592
ses3						
SES	.	.	.	0.049	1	0.8253
_cons	.	.	.	0.634	1	0.4259
ses4						
SES	.	.	.	1.945	1	0.1632
_cons	.	.	.	1.149	1	0.2838
var(e.ses1)	0.189	1	0.6640	.	.	.
var(e.ses2)	0.063	1	0.8023	.	.	.
var(e.ses3)	1.011	1	0.3146	.	.	.
var(e.ses4)	0.090	1	0.7641	.	.	.
var(e.math)	0.065	1	0.7982	.	.	.
var(e.ratio)	36.627	1	0.0000	.	.	.
var(SES)	0.042	1	0.8383	.	.	.

To test group differences in each direct path, we can use the postestimation command `estat ginvariant`. These results show us Wald tests evaluating constraining parameters that were allowed to vary across groups and score tests evaluating relaxing constraints. Both are testing whether individual paths significantly differ across groups.

2.3 Growth Curve Modeling

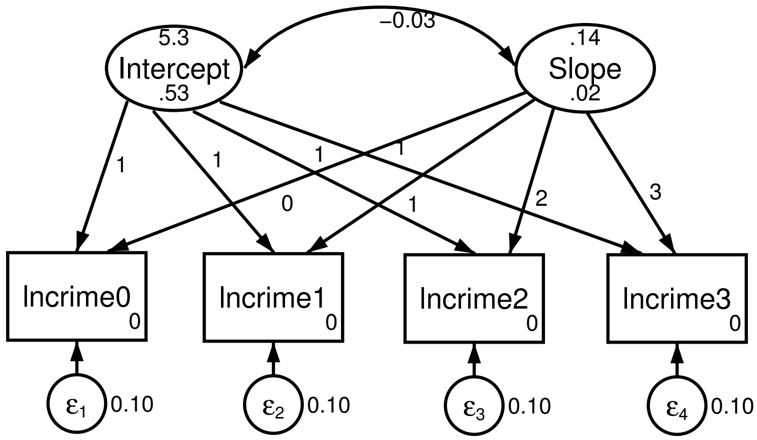
The last model we will fit using `sem` is a growth curve model. This will require a new dataset.

```
. use crime
```

```
. describe

Contains data from crime.dta
 Observations:           359
    Variables:             4                  4 Oct 2012 16:22
                                              (_dta has notes)

Variable      Storage   Display    Value
    name         type    format    label      Variable label

lncrime0        float   %9.0g                 ln(crime rate) in Jan & Feb
lncrime1        float   %9.0g                 ln(crime rate) in Mar & Apr
lncrime2        float   %9.0g                 ln(crime rate) in May & Jun
lncrime3        float   %9.0g                 ln(crime rate) in Jul & Aug

Sorted by:
```

These data are from Bollen and Curran (2006); they contain crime rates collected in two-month intervals for the first eight months of 1995 for 359 communities in New York state. We would like to fit a linear growth curve to these data to model how crime rate changed over time. In our model, we can set constraints using the `@` symbol as we did before. To constrain all intercepts to 0, we can add the `nocons` option. We will also need the `means()` option. By default, Stata constrains the means of latent variables to 0. For this model, we would like to estimate them so we need to specify the latent variable names inside `means()`. We may also consider constraining all the residual variances to equality by constraining each of them to the same arbitrary letter or word, in this case `eps`. See the model in Figure 8.

The estimated mean log crime rate at the beginning of the study was 5.33 and it increased by an average of 0.14 every two months. We could have fit this same model using `gsem`. One way we can do this is to simply replace `sem` with `gsem` in the command in Figure 8. Alternatively, we can can think of this as a multilevel model, and fit it using `gsem`'s notation for random effects. Let's do that next.




```
. sem (Intercept@1 -> lncrime0-lncrime3) (Slope -> lncrime0@0 lncrime1@1
lncrime2@2 lncrime3@3), nocons means(Intercept Slope)
var(e.lncrime0-lncrime3@eps)
```

Figure 8. Growth curve model on crime rate.

3 Fitting models with the gsem command

3.1 Models with Random Effects

The gsem command implements generalizations to the standard linear structural equation model implemented in sem, such as models with generalized-linear response variables, random effects, and categorical latent variables (latent classes). Its syntax is the same as sem, with some different options and postestimation commands. We will start by fitting a random-slope model to the crimes dataset, reproducing the results we obtained with the growth curve model using sem. First, we need to create an observation identification variable and reshape the data into long format.

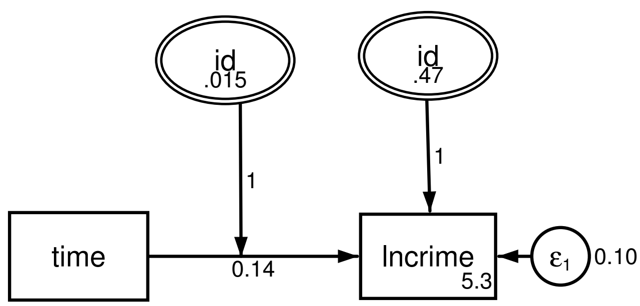
```
. gen id = _n
. reshape long lncrime, i(id) j(time)
(j = 0 1 2 3)
Data                               Wide   ->   Long
-----------------------------------------------------------------------------
Number of observations              359   ->   1,436
Number of variables                   5   ->   3
j variable (4 values)                     ->   time
xij variables:
         lncrime0 lncrime1 ... lncrime3   ->   lncrime
-----------------------------------------------------------------------------
```

```
. summarize
    Variable |        Obs        Mean    Std. dev.       Min        Max
-------------+---------------------------------------------------------
          id |      1,436         180    103.6701          1        359
        time |      1,436         1.5    1.118423          0          3
     lncrime |      1,436    5.551958    .7856259   2.415164   9.575166
```

We now have long-format data in which we have several rows of observations for each individual; we're ready to fit our random-slope model. We specify random effects in `gsem` by adding brackets enclosing the clustering variable to the latent variable, i.e. `Intercept[id]`. This tells Stata to include a latent variable in the model called `Intercept` that has variability at the `id` level. As with other latent variables, it will have a mean of 0 and an initial factor loading of 1, so the only parameter this term introduces is a level-2 variance. Random coefficients can be added to any term by interacting a latent random effect with that variable, i.e. `c.time#Slope[id]`.

Interactions in Stata are specified using `#`; interaction terms are assumed to be factor variables unless prefixed by `c.` to indicate that they are continuous variables. Contrarily, main-effect terms are assumed to be continuous unless prefixed by `i.` to indicate that they are factor variables. We'll see this in the next example. This factor variable notation is not available using `sem`.

See the syntax and results of the random slope model in Figure 9; these results replicate those by `sem`. In the SEM Builder, random effects are represented as double-bordered ovals labeled with the clustering variable to indicate that they represent variability at the cluster level.




```
. gsem (Intercept[id] time c.time#Slope[id] -> lncrime)
```

Figure 9. Random-slope model on crime rate.

3.2 Models with Generalized Responses

The `gsem` command can also be used to fit generalized linear SEMs; that is, SEMs in which an endogenous variable is distributed according to some distribution family and is related to the linear prediction of the model through a link function. See Table 2 for a list of available distribution families and links. Either the family and link can be specified, i.e. `family(bernoulli) link(logit)`, or some combinations have shortcuts that you can specify instead, i.e. `logit`. For this example, we will return to the first dataset.

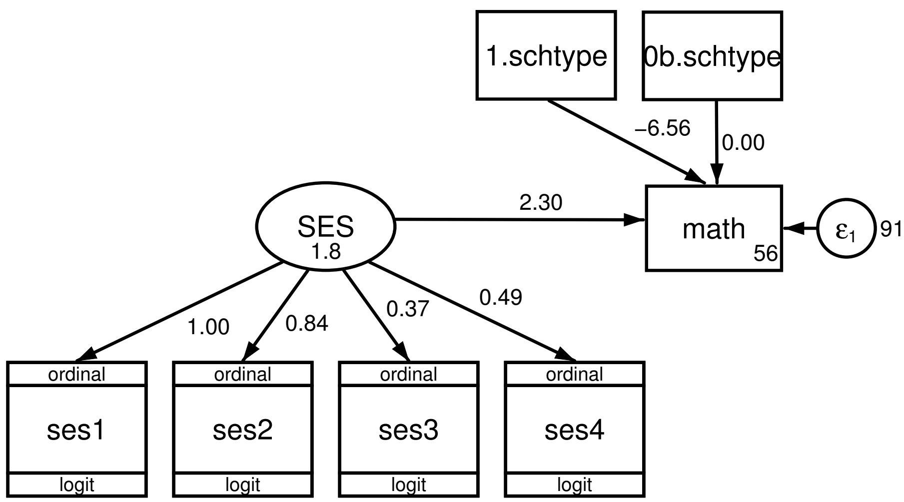
Table 2. `gsem` distribution families and link functions

`family()` options	`link()` options				
	`identity`	`log`	`logit`	`probit`	`cloglog`
`gaussian`	X	X			
`bernoulli`			`logit`	`probit`	`cloglog`
`beta`			X	X	X
`binomial`			X	X	X
`ordinal`			`ologit`	`oprobit`	`ocloglog`
`multinomial`			`mlogit`		
`Poisson`		`poisson`			
`negative binomial`		`nbreg`			
`exponential`		`exponential`			
`Weibull`		`weibull`			
`gamma`		`gamma`			
`loglogistic`		`loglogistic`			
`lognormal`		`lognormal`			

Note: X indicates possible combinations. Where applicable, regression names that imply that family/link combination are shown. If no family/link are provided, `family(gaussian) link(identity)` is assumed.

```
. use math
. codebook, compact
Variable   Obs Unique      Mean  Min  Max  Label
------------------------------------------------------------------------------
schtype    519      2    .61079    0    1  School type
ratio      519     14  16.75723   10   28  Student-Teacher ratio
math       519     42  51.72254   30   71  Math score
ses1       519      5  1.982659    0    4  SES item 1
ses2       519      5  2.003854    0    4  SES item 2
ses3       519      5  2.003854    0    4  SES item 3
ses4       519      5  2.003854    0    4  SES item 4
------------------------------------------------------------------------------
```

In our previous analysis, we had treated each socioeconomic status Likert item as continuous. Now, we will treat them as ordinal using `gsem`. Adding the `ologit` option will fit the measurement model using the ordinal family with a logistic link. We will also use factor variable notation to include indicator variables for school type in our analysis. See figure Figure 10. By adding `schtype` as a factor variable, a dummy variable for each level of `schtype` is included in the model. The path coefficient for the base level, by default the lowest, is constrained to zero. To get exponentiated coefficients, we can follow with the postestimation command `estat eform`.




```
. sem (SES -> ses1-ses4, ologit) (SES i.schtype -> math)
```

Figure 10. Ordinal logistic regression model.

```
. estat eform ses1 ses2 ses3 ses4
```

	exp(b)	Std. err.	z	P>|z|	[95% conf.	interval]
ses1						
SES	2.718282	(constrained)				
ses2						
SES	2.311549	.483485	4.01	0.000	1.534141	3.482899
ses3						
SES	1.449492	.180061	2.99	0.003	1.136257	1.849077
ses4						
SES	1.628133	.2474222	3.21	0.001	1.208748	2.193029

4 Conclusion

In this tutorial, we've shown the basics of fitting SEMs in Stata using the `sem` and `gsem` commands, and have provided example datasets and syntax online to follow along. We demonstrated confirmatory factor analysis, mediation, group analysis, growth curve modeling, and models with random effects and generalized responses. However, there are many possibilities and options not included in this tutorial, such as latent class analysis models, nonrecursive models, reliability models, mediation models with generalized responses, multivariate random-effects models, and much more. Visit Stata's documentation to see all the available options for these commands, their methods and formulas, and many more examples online at `https://www.stata.com/manuals/sem.pdf`.

References

Bollen, K. A., & Curran, P. J. (2006). *Latent curve models: A structural equation perspective* (Vol. 467). John Wiley & Sons.

Satorra, A., & Bentler, P. M. (1994). Corrections to test statistics and standard errors in covariance structure analysis. In *Latent variables analysis: Applications for developmental research.* (pp. 399–419). Sage Publications, Inc.

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